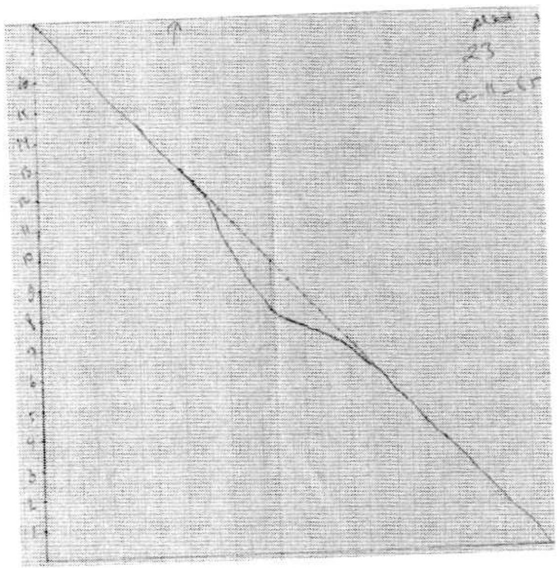
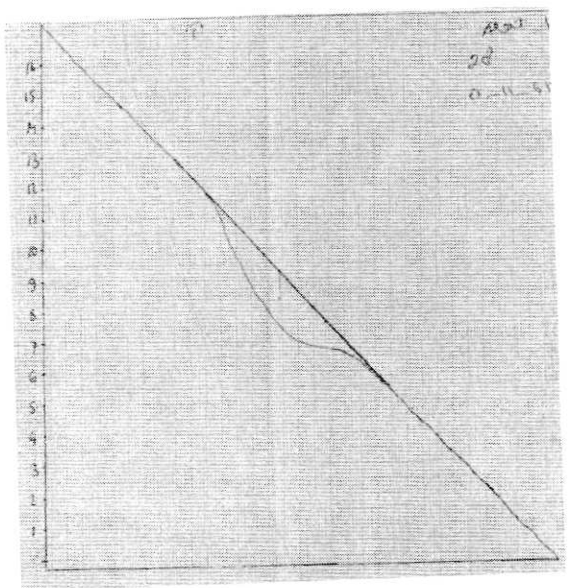
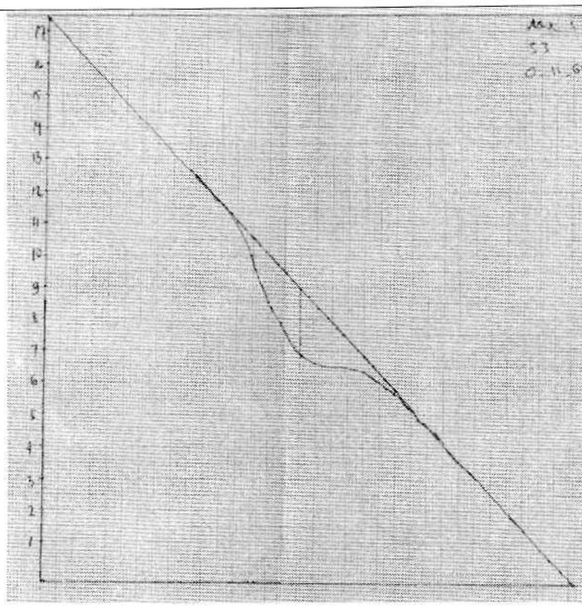
Determination of Refractive Index Gradient and Diffusion Coefficient of Salt Solution From Laser Deflection Measurement (10 points)

A. Measurement of Refractive Index Gradient of Salt Water Solution (4.5 points)

Question	Answer	Marks
A1. (1.2 pts)		Deflactogram of C_0 = 23 g/150 mL # of data = 20
A1.		Deflactogram of C_0 = 28 gr/300 mL # of data = 20

A1.



Deflactogram of C_0 = 33 g/300 mL

A2. **(1.5 pts)**

Table 1 of C_0 = 23 g/150 mL

Optimum Z and Z_0

data = 20

i	δ_i (cm)	ξ_i (cm)	Z_0 (cm)	*d(cm)*	*Z (cm)*
1	0.05	11.55	10.4 ± 0. 1	0.8 ± 0.1	53.4 ± 0.1
2	0.35	11.3			
3	0.6	11.05			
4	0.9	10.85			
5	1	10.65			
6	1.1	10.35			
7	1.3	10.15			
8	1.4	9.85			
9	1.45	9.7			
10	1.5	9.45			
11	1.6	9.25			
12	1.5	8.95			
13	1.4	8.65			
14	1.2	8.35			
15	1	8.05			
16	0.8	7.75			
17	0.7	7.55			
18	0.5	7.25			
19	0.3	6.95			
20	0.2	6.65			
21	0.05	6.4			

A2.

Table 1 of C_0 = 28 g/150 mL

i	δ_i (cm)	ξ_i (cm)	Z_0 (cm)	*d(cm)*	*Z (cm)*
1	0.05	11.65	10.4 ± 0. 1	0.8 ± 0.1	53.4 ± 0.1
2	0.25	11.4			

	3	0.4	11.2				Optimum Z and Z_0
	4	0.8	11				
	5	1	10.75				# data = 20
	6	1.2	10.4				
	7	1.4	10.2				
	8	1.5	10				
	9	1.6	9.8				
	10	1.7	9.5				
	11	1.75	9.25				
	12	1.7	8.95				
	13	1.65	8.7				
	14	1.5	8.4				
	15	1.25	8.05				
	16	0.9	7.6				
	17	0.6	7.3				
	18	0.4	7.05				
	19	0.25	6.75				
	20	0.05	6.3				

A2.

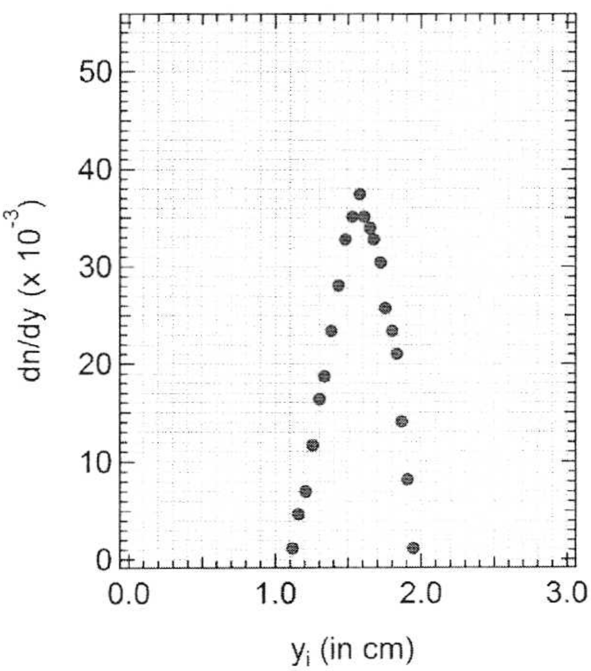
Table 1 of C_0 = 33 g/150 mL

i	δ_i (cm)	ξ_i (cm)	Z_o (cm)	d(cm)	Z (cm)
1	0.05	11.6	10.4 ± 0. 1	0.8 ± 0.1	53.4 ± 0.1
2	0.15	11.4			
3	0.35	11.1			
4	0.65	10.85			
5	1.1	10.6			
6	1.3	10.4			
7	1.5	10.2			
8	1.7	10			
9	1.85	9.7			
10	2	9.5			
11	2.1	9.25			
12	2	9			
13	1.8	8.6			
14	1.5	8.3			
15	1.25	8.05			
16	1	7.8			
17	0.75	7.45			
18	0.55	7.15			
19	0.4	6.8			
20	0.2	6.4			
21	0.05	6.1			

A3. (1.5 pts)		Table 2 of C_0 = 23 g/150 mL. # data = 20

i	Y_i (cm)	dn/dY
1	1.85944	0.00117
2	1.81919	0.00819
3	1.77894	0.01404
4	1.74674	0.02106
5	1.71455	0.02340
6	1.66625	0.02574
7	1.63405	0.03043
8	1.58575	0.03277
9	1.56161	0.03394
10	1.52136	0.03511
11	1.48916	0.03745
12	1.44086	0.03511
13	1.39257	0.03277
14	1.34427	0.02809
15	1.29597	0.02340
16	1.24767	0.01872
17	1.21548	0.01638
18	1.16718	0.01170
19	1.11888	0.00702
20	1.07058	0.00468
21	1.03034	0.00117

A3.



Plot dn/dY vs Y C_0 = 23 g/150 mL.

"Gaussian-Like" shape

A3.

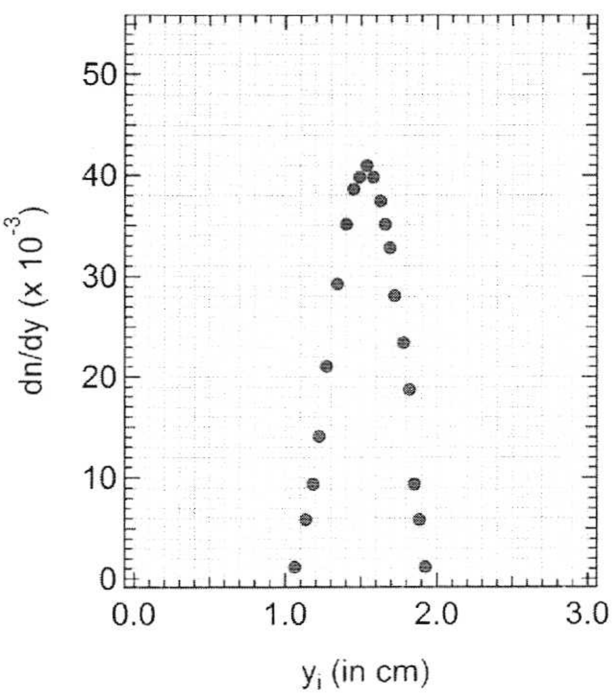
Table 2 of C_0 = 28 g/150 mL.

i	Y_i (cm)	dn/dY
1	1.87554	0.00117

2	1.83529	0.00585
3	1.80309	0.00936
4	1.77089	0.01872
5	1.73065	0.02340
6	1.67430	0.02809
7	1.64210	0.03277
8	1.60990	0.03511
9	1.57770	0.03745
10	1.52941	0.03979
11	1.48916	0.04096
12	1.44086	0.03979
13	1.40061	0.03862
14	1.35232	0.03511
15	1.29597	0.02926
16	1.22352	0.02106
17	1.17523	0.01404
18	1.13498	0.00936
19	1.08668	0.00585
20	1.01424	0.00117

data = 20

A3.



Plot dn/dY vs Y
C_0 = 28 g/150 mL.

"Gaussian-Like" shape

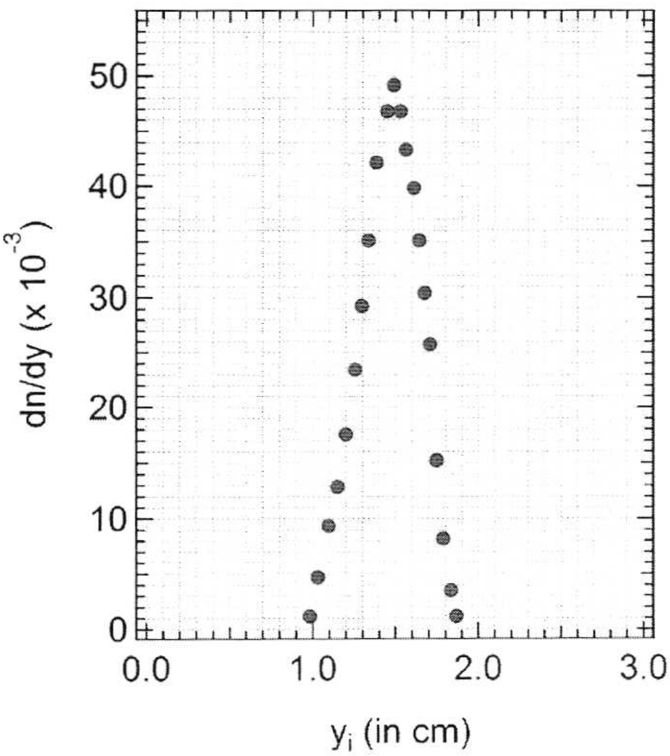
A3.

i	Y_i (cm)	dn/dY
1	1.86749	0.00117
2	1.83529	0.00351

Table 2 of C_0 = 33 g/150 mL.

3	1.78699	0.00819	# data = 20
4	1.74674	0.01521	
5	1.70650	0.02574	
6	1.67430	0.03043	
7	1.64210	0.03511	
8	1.60990	0.03979	
9	1.56161	0.04330	
10	1.52941	0.04681	
11	1.48916	0.04915	
12	1.44891	0.04681	
13	1.38452	0.04213	
14	1.33622	0.03511	
15	1.29597	0.02926	
16	1.25572	0.02340	
17	1.19938	0.01755	
18	1.15108	0.01287	
19	1.09473	0.00936	
20	1.03034	0.00468	
21	0.98204	0.00117	

A3.



Plot dn/dY vs Y
C_0 = 33 g/150 mL.

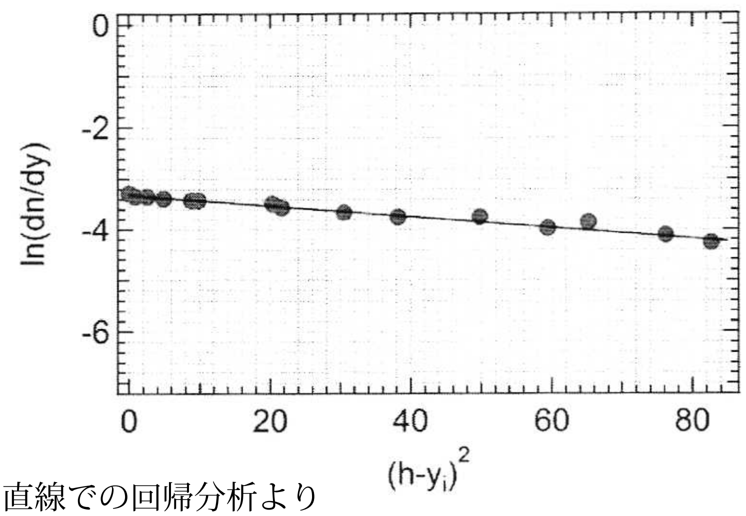
A4. **(0.3 pts)** *h* for 23 g/ 300 mL = (1.5 $\pm$ 0.1) cm

	h for 28 g/ 300 mL = (1.5 ± 0.1) cm *h* for 33 g/ 300 mL = (1.5 ± 0.1) cm	

B : Determination of Diffusion Coefficient (4.2 points)

Question	Answer	Marks
B1. **(0.9 pts)**	**Linear form of eq.(3)** $\ln\left(\frac{dn}{dY}\right) \approx m(h-Y)^2 + C$ (b1.1) 横軸(Abscissa) Absisca : $(h-y)^2$ (b1.2) 縦軸 Ordinate : $\ln\left(\frac{dn}{dY}\right)$ (b1.3) 傾き Gradient : $m = -\frac{1}{4D_e t}$ (b1.4) 定数（y切片） Constant : $C = \ln\left(\left(\frac{dn}{dc}\right)\left(\frac{C_0}{2\sqrt{\pi D_e t}}\right)\right)$ (b1.5)	
B2. **(1.8 pts)**	(see table below)	Table 3 of C_0 = 23 g /150 mL. # data = 10
B2		Plot of Table 3

i	(h-yi)^2	ln(dn/dy)
1	0.06592	-3.86003
2	0.050423	-3.75467
3	0.031065	-3.65936
4	0.020752	-3.4923
5	0.00917	-3.41819
6	0.005128	-3.3831
7	0.000984	-3.3492
8	6.99E-07	-3.28466
9	0.002414	-3.3492
10	0.009493	-3.41819
11	0.021237	-3.57235
12	0.037646	-3.75467
13	0.05872	-3.97781



直線での回帰分析より
Using linear regression of eq. (B1.1), we obtain

傾き m (gradient) = -9.4 ± 0.5 $cm^{-2}\,s^{-1}$

C_0 = 23 g/150 mL

data = 10

B2.

i	$(h\text{-}yi)^2$	ln(dn/dy)
1	0.057912	-3.75467
2	0.033968	-3.57235
3	0.023136	-3.41819
4	0.014378	-3.3492
5	0.007693	-3.28466
6	0.001553	-3.22404
7	6.99E-07	-3.19505
8	0.002414	-3.22404
9	0.007989	-3.25389
10	0.018955	-3.3492
11	0.037646	-3.53152
12	0.071007	-3.86003
13	0.099079	-4.26549

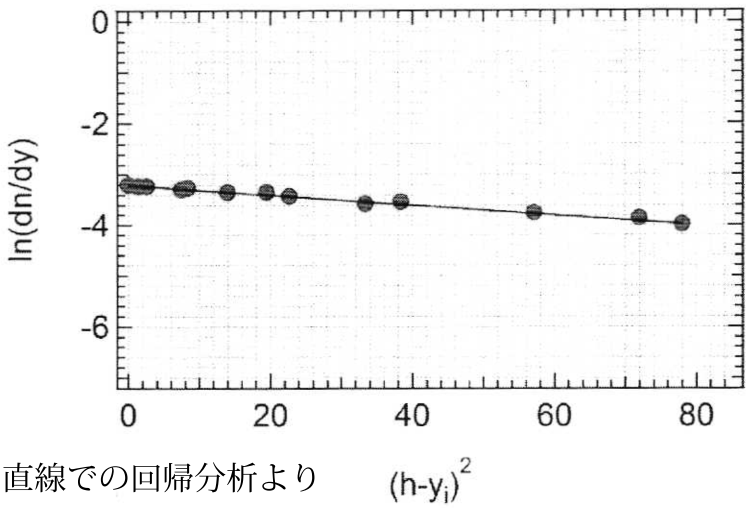
Table 3 of C_0 = 28 g /150 mL

data = 10

B2.

Plot of Table 3 C_0 = 28 g/150 mL

data = 10



直線での回帰分析より

Using linear regression of eq. (B1.1), we obtain

傾き

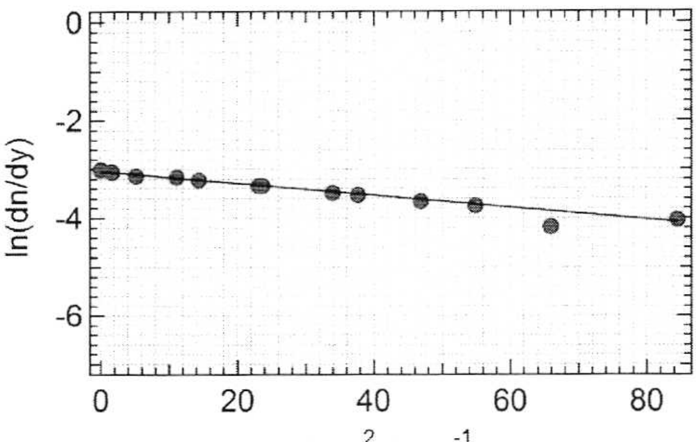
m (gradient) = −10.3 ± 0.5 $cm^{-2}\ s^{-1}$

B2.

i	$(h-yi)^2$	ln(dn/dy)
1	0.046873	-3.65936
2	0.033968	-3.4923
3	0.023136	-3.3492
4	0.014378	-3.22404
5	0.005128	-3.13948
6	0.001553	-3.06152
7	6.99E-07	-3.01273
8	0.001688	-3.06152
9	0.011126	-3.16688
10	0.023647	-3.3492
11	0.037646	-3.53152
12	0.054884	-3.75467
13	0.08446	-4.04235

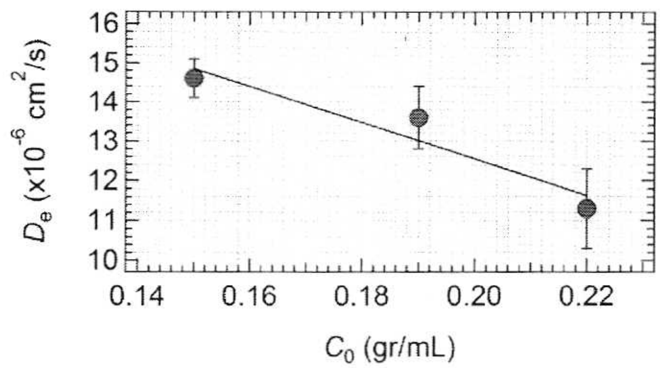
Table 3 of C_0 = 33 g /150 mL

data = 10

B2. 傾き	 直線での回帰分析より Using linear regression of eq. (B1.1), we obtain *m* (gradient) = −12.3 ± 0.5 $cm^{-2}\ s^{-1}$	Plot of Table 3 C_0 = 33 g/150 mL # data = 10
B3 **(1.5 pts)**	Using eq. (B1.4), we obtain D_e of 23 g/ 150 mL = (1.48 ± 0.08) x 10^{-5} cm^2/s D_e of 28 g/ 150 mL = (1.36 ± 0.05) x 10^{-5} cm^2/s D_e of 33 g/ 150 mL = (1.13 ± 0.09) x 10^{-6} cm^2/s	

C. Determination of the rate change of the diffusion coefficient to the change of salt solution concentrations (1.3 points)

Question	Answer	Marks
C1. **(1.3 pts)**		Plot D_e vs. C_0

		
C1.	直線での回帰分析より，食塩水の濃度に対する拡散係数の変化率は Using linear regression we obtain ther ate change of diffusion coefficient w.r.t the change of salt solution concentrations: $$\frac{d}{dC}D_e = (-4.6 \pm 1.4) \times 10^{-5} cm^2\ mL\ gr^{-1} s^{-1}$$	