General Grading Guidelines

When student's solutions are correct and s/he also show how solutions were obtained, the stduent gets full credit. The scheme oulined below is helpful if the student's answers are partially correct. Attention will be paid to the detailed solution so, if the final answer is correct but it is obtained by incorrect method(s) then no credit will be given. Alternative solutions may exist and will be given due credit.

Partial or full outcomes obtained for later sections in the problem which are incorrect solely because of errors being carried forward from previous sections, but are otherwise reasonable, will not be further penalized. For example a dimensioanlly wrong answer when carried forward will not get any credit in the subsequent sections. A numerically wrong evaluation when carried forward will get credit in subsequent sections unless the numerical answer is patently wrong (e.g. the value of g is 981 m/sec^2!)

Incorrect or no labeling of an axis is penalized by -0.1 points

The numerical answer (i) must be correct to +/- 10% AND (ii) must respect significant figures.

It maybe noted that NO micro-marking scheme takes care of all contingencies. A certain amount of discretion rests with and a certain level of judgement is invested in the academic committee.

The Stern-Gerlach Experiment: THE SOLUTION

A.1 Speed of the Silver Atoms: 0.5pt

We employ the equipartition theorem. Let $\overline{v^2}$ be the mean square speed of the silver atoms in the oven kept at 1200 K. Then

$$\frac{m\overline{v^2}}{2} = \frac{3k_BT}{2}$$

where k_B is the Boltzmann constant. This yields the root mean square speed to be 5.255×10^2 m·s^{-1}.

[0.5]

B.1 **The Basic Expression** 2pt

The length l_1 is irrelevant and will not be part of the expression.
The magnitude of the acceleration a of the silver atoms in the region defined by l_2 is

$$a = \frac{\mu_s}{m}\frac{dB}{dx}$$

[0.4]

and it will be either in the +x or -x direction. At the same time it has a constant horizontal velocity v_z. It traverses the region l_2 in time l_2/v_z. Thus after traversing the inhomogeneous region the deflection in say the +x direction is

$$\delta_1 = \frac{1}{2}\frac{\mu_s}{m}\frac{dB}{dx}\frac{l_2^2}{v_z^2}$$

[0.6]

For the remaining part of the flight the atom will have a constant hoirzontal speed v_z and a constant vertical speed $v_{x0} = (\mu_s dB/dx)(l_2/mv_z)$. On account of the v_x component the atom will acquire an additional deflection

$$\delta_2 = l_3 v_{x0}/v_z$$

This yields

$$\delta_2 = l_3 l_2 \frac{\mu_s}{mv_z^2}\frac{dB}{dx}$$

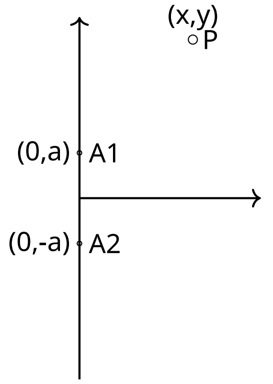
[0.4]

The total deflection in the +x direction is $\delta_1 + \delta_2$. The splitting seen on the screen in this idealized case is twice this amount, e.g. 2 $(\delta_1 + \delta_2)$. Thus we obtain

$$\Delta x = 2\frac{\mu_s}{m}\frac{dB}{dx}\frac{l_2}{v_z^2}(l_2/2 + l_3)$$

[0.6]

C.1 **The Inhomogeneous Magnetic Field:** 1.5pt



Let

$$\vec{A_1P} = \vec{r}_1 = x\hat{i} + (y-a)\hat{j}$$

and

$$\vec{A_2P} = \vec{r}_2 = x\hat{i} + (y+a)\hat{j}$$

This gives for $\vec{B}(x,y)$

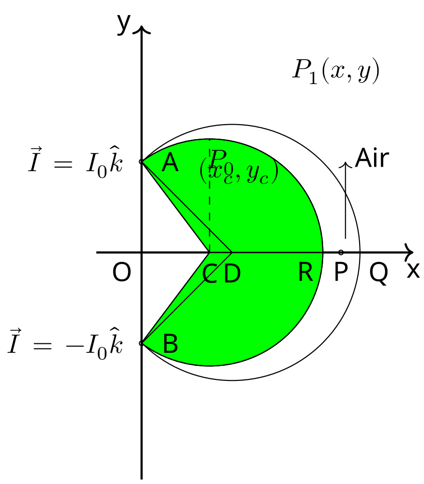
$$\frac{\mu I_0}{2\pi}\left[\frac{\hat{k}\times(x\hat{i}+(y-a)\hat{j}}{r_1^2} - \frac{\hat{k}\times(x\hat{i}+(y+a)\hat{j}}{r_2^2}\right] \quad (1)$$

[0.4+0.4]

$$= \frac{\mu I_0}{2\pi r_1^2 r_2^2}\left[(x\hat{j}-(y-a)\hat{i})(x^2+(y+a)^2) - (x\hat{j}-(y+a)\hat{i})(x^2+(y-a)^2)\right]$$

$$= \frac{\mu I_0\, a}{\pi r_1^2 r_2^2}\left[2xy\hat{j} + (x^2-y^2+a^2)\hat{i}\right] \quad (2)$$

[0.7]



C.2 **Direction at point** R: Field at the point R $((x_c + \sqrt{x_c^2 + a^2}, 0)$ is given by substituting $y = 0$. On simple inspection the $\hat{j}$ component vanishes. Thus $\vec{B}(x, 0) \propto \hat{i}$ — 0.5pt

[0.2]

Direction at point P_0: Field at P_0 $((x_c, y_c = (x_c^2 + a^2)^{1/2}))$ is given, using Eq.(2)

$$\frac{\mu I_0}{\pi r_1^2 r_2^2}(2x_c(x_c^2 + a^2)^{1/2}\hat{j} + (x_c^2 - x_c^2 - a^2 + a^2)\hat{i})$$

The $\hat{i}$ component is zero. Thus $\vec{B}(x_c, (x_c^2 + a^2)^{1/2}) \propto \hat{j}$

[0.3]

First Alternative Solution

We can show in general that the field at **any** point on the circle will be radial (i.e. normal to the circle). We will confine our discussion to the z=0 plane. Consider a point (x_c, y) with radius $\sqrt{x_c^2 + a^2}$. The equation of a circle with $(x_c, 0)$ as centre and $\sqrt{x_c^2 + a^2}$ as radius is

$$(x - x_c)^2 + y^2 = x_c^2 + a^2$$

or

$$x^2 - 2\,x\,x_c + y^2 = a^2 \tag{3}$$

If at the point (x_c, y_c) the magnetic field is along $\hat{j}$ then the component along $\hat{i}$ is zero. $(x_c, 0)$ is identified with the point C on the figure. The point y_c is then,

$$x_c^2 - y_c^2 + a^2 = 0$$

or

$$y_c^2 = x_c^2 + a^2 \tag{4}$$

Now consider a line joining C $(x_c, 0)$ to any point $P_C(x, y)$ lying on the circle given by eq.(3). The radial vector is $\vec{CP}_C = (x - x_c)\hat{i} + y\hat{j}$. The magnetic field at P_C is

$$\propto \vec{B}(x, y, 0) = \left(\frac{\mu I_0}{\pi}\right)\left(2xy\hat{j} + (x^2 - y^2 + a^2)\hat{i}\right)$$

To show that they are in the same direction, we evaluate the cross product, $\vec{CP}_C \times \vec{B}$. The cross product is proportional to $\hat{n}$ which is a unit vector along the direction which is normal to both CP_C and $\vec{B}$ and is along $\hat{k}$

$$\vec{CP}_C \times \vec{B} \propto (2\,x\,y(x - x_c) - y(x^2 - y^2 + a^2))\,\hat{k}$$

which simplifies to

$$y(x^2 - 2x\,x_c + y^2 - a^2)\hat{k}$$

Using eq.(3), this is zero, proving the result.

C.2 (cont.)

Second Alternative Solution
To show that the field lines are radial over the circe one may merely show the proportionality of the components of the field and the radius vector. The radius vector is $(x - x_c)\hat{\imath} + y\hat{\jmath}$ while the magnetic field is proportional to $(x^2 - y^2 + a^2)\hat{\imath} + 2xy\hat{\jmath}$. Thus

$$\frac{y}{2xy} = \frac{1}{2x}$$

and

$$\frac{x - x_c}{x^2 - y^2 + a^2} = \frac{1}{2x}$$

The last step is obtained by observing that the equation of the circle is $(x - x_c)^2 + y^2 = x_c^2 + a^2$.

C.3 Field in the airgap because of the argument presented in the problem continues to be given by Eq.(2). So the field ($y = 0$), is again — 0.5pt

$$\vec{B} = \frac{\mu I_0 \, a}{\pi(x^2 + a^2)}\hat{\imath}$$

[0.5]

D.1 The force F_x on a magnetic dipole along the x- direction is — 0.5pt

$$F_x = -\mu_s \frac{\partial B_x}{\partial x} = \frac{\mu_s \mu I_0}{\pi} \times \frac{2\,a\,x}{(x^2 + a^2)^2} \qquad (5)$$

[0.5]

E.1 2.0pt

$$\frac{\mu}{\mu_0} = 10^4; \quad a = 6.00\times10^{-3}m; \quad \text{OC} = 6.00\times10^{-3}m; \quad \text{OD} = 8.00\times10^{-3}m;$$

and

$$I_0 = 2.00\,A$$

and so at the midpoint P,

$$y = 0;$$

$$x_P = OP = ((1 + \sqrt{2}) \times .6 + 1.8)/2 = 1.624 \times 10^{-2}m$$

[0.5]

where we have used $OD = .8\times10^{-2}m$ and $DA = 10^{-2}m$. This gives for $B_x(x_P, 0)$

$$\frac{\mu}{\mu_0}\frac{\mu_0}{\pi}\frac{I_0\,a}{(x_P^2 + a^2)} = \frac{10^4 \times 4 \times 10^{-7} \times 2 \times .6 \times 10^{-2}}{(1.624^2 + .6^2) \times 10^{-4}}$$

$$= 0.16\,T$$

[1]

We also have

$$\left(\frac{\partial B_x}{\partial x}\right)_{x_P} = \frac{2 \times x_p}{x_p^2 + a^2} \times B_x(x_P, 0) = \frac{2 \times 1.624 \times 10^{-2}}{(1.624^2 + .6^2) \times 10^{-4}} \times .16 = 17.34\,T \cdot m^{-1}$$

[0.5]

F.1 **The magnetic moment of the silver atom:** 1.5pt

We use

$$\Delta x = \frac{2\mu_s}{m}\left(\frac{\partial B}{\partial x}\right)_{x_P}\frac{l_2}{v_z^2}\left(\frac{l_2}{2} + l_3\right)$$

to rewrite

$$\mu_s = \frac{m\Delta x}{2(\frac{\partial B_x}{\partial x})_{x_P}} \times \frac{1}{[\frac{l_2}{v_z^2}(\frac{l_2}{2} + l_3)]}$$

[0.5]

$$= \frac{1.8 \times 10^{-25} \times 2 \times 10^{-3}}{2 \times 17.34} \times 10^6 = 1.04 \times 10^{-23} J \cdot T^{-1}$$

[1]

G.1 **The spread in the line:** 0.5pt

The two lines on the screen are separated symmetrically about the centre by Δx. So the upper (lower) line is at $\Delta x/2$ from the centre. From Part (2)

$$\Delta x/2 = \frac{\mu_s}{m}\frac{dB}{dx}\frac{l_2}{v_z^2}(l_2/2 + l_3)$$

This depends on the beam speed v_z. The spread in this speed leads to a consequent spread in the splitting.

$$\begin{aligned}\delta(\Delta x/2) &= |\frac{\partial \Delta x/2}{\partial v_z}|\delta v_z \\ &= 2(\Delta x/2)\frac{\delta v_z}{v_z} \\ &= 2(\Delta x/2)\times 0.2 \\ &= 0.04 cm\end{aligned}$$

[0.3]

Hence the spread in the line from the centre is 0.1 - 0.04 = 0.06 cm to 0.1 + 0.04 = 0.14 cm.

[0.2]

H.1 **Error in the evaluation of the magnetic moment:** 0.5pt

From the previous part we have that the splitting ranges from 0.12 cm to 0.28 cm whereas earlier it was 0.2 cm. The relationship between the splitting and the magnetic moment is linear. So the magnetic moment ranges from (0.12/0.2) to (0.28/0.2) the original value. This yields 0.62 $\times$ 10^{-23} J·T^{-1} to 1.46 $\times$ 10^{-23} J·T^{-1}. The total spread is 0.84 $\times$ 10^{-23} J·T^{-1} about the mean value of 1.04 $\times$ 10^{-23} J·T^{-1}

[0.3]

or in other words

$$\mu_s = 1.04 \pm 0.42\,\text{J·T}^{-1}$$

[0.2]